



Der Wissenschaftsfonds.



Institute of Analysis and Number Theory

Zahlentheoretisches Kolloquium

Friday, 19.7.2019, 14:30

Seminarraum Analysis-Zahlentheorie, Kopernikusgasse 24, 2.OG

Finiteness results on a certain class of modular forms and applications

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Holomorphic eta quotients are certain explicit classical modular forms on suitable Hecke subgroups of the full modular group. We call a holomorphic eta quotient f 'reducible' if for some holomorphic eta quotient g (other than 1 and f), the eta quotient f/g is holomorphic. An eta quotient or a modular form in general has two parameters: Weight and level.

We shall show that for any positive integer N , there are only finitely many irreducible holomorphic eta quotients of level N . In particular, the weights of such eta quotients are bounded above by a function of N . We shall provide such an explicit upper bound. This is an analog of a conjecture of Zagier which says that for any positive integer k , there are only finitely many irreducible holomorphic eta quotients of weight $k/2$ which are not integral rescalings of some other eta quotients.

This conjecture was established in 1991 by Mersmann. We shall sketch a short proof of Mersmann's theorem and we shall show that these results have their applications in factorizing holomorphic eta quotient. In particular, due to Zagier and Mersmann's work, holomorphic eta quotients of weight $1/2$ have been completely classified. We shall see some applications of this classification and we shall discuss a few seemingly accessible yet longstanding open problems about eta quotients.

This talk will be suitable also for non-experts: We shall define all the relevant terms and we shall clearly state the classical results which we use.

Remark: Soumya Bhattacharya is a new member of the Institute of Analysis and Number Theory. He will stay in Graz for one year as a PostDoc researcher.

Ch. Aistleitner