Institute of Analysis and Number Theory

FWF START Seminar

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Seminarraum Analysis-Zahlentheorie, Kopernikusgasse 24, 2.OG

Some complexity results in the theory of normal numbers

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Let $N(b)$ be the set of real numbers which are normal to base b. A well-known result of H. Ki and T. Linton is that $N(b)$ is $\mathbf{\Pi}^0_3$-complete. We show that the set $N^{\perp}(b)$ of reals which preserve $N(b)$ under addition is also $\mathbf{\Pi}^0_3$-complete. We use the characterization of $N^{\perp}(b)$ given by G. Rauzy in terms of an entropy-like quantity called the *noise*. It follows from our results that no further characterization theorems could result in a still better bound on the complexity of $N^{\perp}(b)$. We compute the exact descriptive complexity of other naturally occurring sets associated with noise. One of these is complete at the $\mathbf{\Pi}^0_4$ level. Finally, we get upper and lower bounds on the Hausdorff dimension of the level sets associated with the noise.

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