Der Wissenschaftsfonds.

Institut f. Analysis und Zahlentheorie

Zahlentheoretisches Kolloquium

Freitag, 11. 3. 2016, 14:00 Uhr

SR NT02008, Kopernikusgasse 24/2.Stock

Algebraic independence results for reciprocal sums formed by powers of Fibonacci numbers

Prof. Dr. Carsten Elsner

(Fachhochschule für die Wirtschaft Hannover)

Abstract:

Let $(U_n)_{n\geq 0}$ be a sequence of numbers given by $U_n = (\alpha^n - \beta^n)/(\alpha - \beta)$ for $n \geq 0$, where α and β are complex numbers satisfying $|\beta| < 1$ and $\alpha\beta = -1$. Let $s \geq 1$ and

$$\Phi_{2s} := (\alpha - \beta)^{-2s} \sum_{n=1}^{\infty} \frac{1}{U_n^{2s}}\,.$$

In a joint paper of the lecturer with Prof. Shun Shimomura and Prof. Iekata Shiokawa it is shown that for any positive integers s_1, s_2, s_3 corresponding to algebraic α, β with $|\beta| < 1$ and $\alpha\beta = -1$ the numbers $\Phi_{2s_1}, \Phi_{2s_2}, \Phi_{2s_3}$ are algebraically independent over if and only if $s_1 s_2 s_3 \equiv 0 \pmod 2$. In particular, for $\beta = (1 - \sqrt{5})/2$ and $\beta = 1 - \sqrt{2}$ the results on Φ_{2s} are applicable to reciprocal sums formed by powers of Fibonacci numbers $U_n = F_n$ and by powers of Pell numbers $U_n = P_n$, respectively. In the lecture the proof of the algebraic independence of $\Phi_{2s_1}, \Phi_{2s_2}, \Phi_{2s_3}$ is presented for the particular case of three even integers $2 \leq s_1 < s_2 < s_3$.

N. Technau, R. Tichy