Institut f. Analysis und Zahlentheorie

Zahlentheoretisches Kolloquium

Freitag, 14. 12. 2018, 15:00 s.t.

Seminarraum Analysis-Zahlentheorie (NT02008), Kopernikusgasse 24/II

On the polynomial Pell equation

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It is well-known that the Pell equation $a^2 - db^2 = 1$ in integers has a non-trivial solution if and only if d is positive and not a perfect square. If one considers the polynomial analog, i.e., for fixed $D \in \mathbb{C}[X]$, the equation $A^2 - DB^2 = 1$, the matter is more complicated. Indeed, for the existence of a solution the clear necessary conditions that the degree of D must be even and that D cannot be a perfect square are not sufficient. While the case of degree two is analogous to the integer case, there are non-square polynomials of degree 4 such that the corresponding Pell equation is not solvable. On the other hand, as in the integer case, once we have a non-trivial solution, we have infinitely many and we call minimal solution a solution (A, B) with A of minimal degree.

In joint work with Laura Capuano and Umberto Zannier we showed that there exist equations $A^2 - DB^2 = 1$, with (A, B) minimal solution, for any choice of degrees $\deg D \geq 4$ even and $\deg A \geq \deg D/2$.

D.Kreso