Der Wissenschaftsfonds.

Institute of Analysis and Number Theory

Number Theory Seminar

Tuesday, 21.5.2019, 11:15.

Seminarraum Analysis-Zahlentheorie, Kopernikusgasse 24, 2.OG.

Khintchine's theorem with extra divergence instead of monotonicity

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New results towards the Duffin-Schaeffer conjecture, which is a fundamental unsolved problem in metric number theory, have been established recently assuming extra divergence. Given a non-negative function $\psi : \mathbb{N} \to \mathbb{R}$ we denote by $W(\psi)$ the set of all $x \in \mathbb{R}$ such that $|nx - a| < \psi(n)$ for infinitely many a, n. Analogously, we write $W'(\psi)$ if we additionally require a, n to be coprime.

Aistleitner et al. proved that $W'(\psi)$ is of full Lebesgue measure if there exists an $\varepsilon > 0$ such that $\sum_{n=2}^{\infty} \psi(n)\varphi(n)/(n(\log n)^{\varepsilon}) = \infty$. This result seems to be the best one can expect from the method used. Assuming the extra divergence $\sum_{n=2}^{\infty} \psi(n)/(\log n)^{\varepsilon} = \infty$ we prove that $W(\psi)$ is of full measure. This could also be deduced from the results in Aistleitner et al., but we believe that our proof is of independent interest, since its method is totally different from theirs. As a further application of our method, we prove that a variant of Khintchine's theorem is true without monotonicity, if the support of ψ can be restricted subject to a condition on the set of divisors.

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