Institut f. Analysis und Zahlentheorie

## Zahlentheoretisches Kolloquium

Freitag, 21. 9. 2018, 14:00 Uhr

Seminarraum Analysis-Zahlentheorie (NT02008), Kopernikusgasse 24/II

# The uniqueness of the extension of infinite two-parameter family of Diophantine triples

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Abstract: A set of $m$ positive integers is called a Diophantine $m$-tuple if the product of any two elements in the set increased by 1 is a perfect square. One of the question of interest is how large those sets can be. Very recently He, Togbé and Ziegler proved the folklore conjecture that there does not exist a Diophantine quintuple.

There is also stronger version of that conjecture which states that every Diophantine triple can be extended to a quadruple, with a larger element, in a unique way. That conjecture is still open. In this talk we study the two families of Diophantine pairs and consider their extension. More precisely we prove the mentioned conjecture for the triples $\{a, b, c\}$, where $a$ and $b$ are positive integers defined by $a = KA^2$, $b = 4KA^4 + 4\varepsilon A$ with $K, A$ positive integers and $\varepsilon \in \{\pm 1\}$ and $c$ is given by $c = c_\nu^\tau$, where

$$c_\nu^\tau = \frac{1}{4ab}\left\{(\sqrt{b} + \tau\sqrt{a})^2(r + \sqrt{ab})^{2\nu} + (\sqrt{b} - \tau\sqrt{a})^2(r - \sqrt{ab})^{2\nu} - 2(a + b)\right\}$$

with $\nu$ a positive integer and $\tau \in \{\pm\}$.

This is joint work with Mihai Cipu and Yasutsugu Fujita.

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