Institut f. Analysis und Zahlentheorie

Zahlentheoretisches Kolloquium

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Seminarraum C 208, 2. Stock, Steyrergasse 30, TU Graz

Divisibility of binomial coefficients by powers of two

Dr. Lukas Spiegelhofer

(TU Wien)

Abstract

It is known that the number $a(\alpha,n)$ of binomial coefficients $\binom{n}{t}$ exactly divisible by 2^α can be expressed using a polynomial P_α in block-additive functions in base 2. For example, we have

$$a(1,n)/a(0,n)\& = \tfrac{1}{2}|n|_{\mathbf{10}},$$
$$a(2,n)/a(0,n)\& = -\tfrac{1}{8}|n|_{\mathbf{10}} + |n|_{\mathbf{100}} + \tfrac{1}{4}|n|_{\mathbf{110}} + \tfrac{1}{8}|n|_{\mathbf{10}}^2,$$

where $|n|_w$ is the number of times the word w occurs as a subword of the base-2 expansion of n.

In this talk, we present a method for obtaining the sequence of coefficients of a given monomial in the polynomials $P_0, P_1, \ldots$ as a generating function.

In particular, we apply this method to the monomial $X_{\mathbf{10}}^k$ (corresponding to the term $|n|_{\mathbf{10}}^k$) and the associated sequence $\left(c_\alpha^{(k)}\right)_{\alpha\geq 0}$ of coefficients, obtaining the generating function

$$\sum_{\alpha\geq 0} c_\alpha^{(k)} x^\alpha = \left(\log\left(1+\frac{x}{2}\right)\right)^k.$$

P. Grabner