Institut f. Analysis und Zahlentheorie

Zahlentheoretisches Kolloquium
ACHTUNG - Zeit und Ort des Vortrages haben sich geändert!

Freitag, 30. 6. 2017, 14:00 s.t.

Seminarraum Analysis-Zahlentheorie (NT02008), Kopernikusgasse 24/II

An invitation to spectral spaces

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Prime ideals and prime spectra of rings play a central role both in Commutative Ring Theory and Algebraic Geometry, being them foundation of Scheme Theory, for instance. Order properties and topological properties of spaces of prime ideals have been useful to characterize some classes of rings. From the topological point of view, since the 60s it has been of interest to put in evidence conditions that a topological space must satisfy in order to be homeomorphic to the prime spectrum of some (commutative) ring (with 1). This was the main subject of the PhD thesis of M. Hochster, where he proved that the topological spaces that are homeomorphic to the prime spectrum of a ring - also called *spectral spaces* - are precisely the spaces X satisfying the following axioms:

1. X is compact;
2. X admits a basis of open and compact subspaces that is closed under finite intersections.
3. X is *sober*, that is, every irreducible closed subspace of X has a unique generic point.

While for some class of spectral spaces, like Riemann-Zariski spaces (see [2]), a class of rings realizing Hochster corrispondence was explicitly found, for several other spaces naturally arising in Commutative Ring Theory it is non trivial to understand if they are spectral because, in particular, it can be not so easy to verify condition (3) of Hochster's characterization.

A goal of this survey talk is to present some new perspective about the study of spectral spaces and, in particular, a criterion, based on ultrafilters, to decide if a topological space is spectral (see [1]). Some recent new examples will be discussed.

References

[1] C. A. Finocchiaro, Spectral spaces and ultrafilters. *Comm. Algebra* **42** 2014, no. 4, 1496–1508.

[2] C. A. Finocchiaro, M. Fontana, K. A. Loper, The constructible topology on spaces of valuation domains. *Trans. Amer. Math. Soc.* **365** 2013, no. 12, 6199–6216.

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