Analysis und Zahlentheorie

Zahlentheoretisches Kolloquium

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POTENTIAL THEORY WITH MULTIVARIATE KERNELS

RYAN MATZKE

(University of Minnesota)

In the past century, a great deal of study has been devoted to energy optimization problems. Given a compact metric space Ω and a kernel $K : \Omega^2 \to \mathbb{R} \cup \{\infty\}$, these problems involve finding points sets $\omega_N = \{z_1, ..., z_N\}$ in Ω or Borel probability measures μ on Ω that minimize/maximize the discrete energy

$$E_K(\omega_N) = \frac{1}{N^2} \sum_{j,k=1}^{N} K(z_j, z_k)$$

or the continuous energy

$$I_K(\mu) = \int_\Omega \int_\Omega K(x,y) d\mu(x) d\mu(y),$$

respectively. Such optimization problems can have applications in signal processing, discrepancy theory, discretization of manifolds, discrete geometry, and models of physical phenomena, a classical example coming from the Thomson problem from 1904, which asks what configurations minimize the electrostatic potential energy of N electrons on the sphere.

While a wealth of theory has been developed for classical energies, which model pairwise interactions between particles, of the above forms, there as been little to no systematic study of multivariate energies, defined by interatctions of triples, quadruples, or even higher numbers of particles, i.e. energies of the type

$$E_K(\omega_N) = \frac{1}{N^n} \sum_{j_1,...,j_n=1}^{N} K(z_{j_1}, ..., z_{j_n})$$

and

$$I_K(\mu) = \int_\Omega \cdots \int_\Omega K(x_1, ..., x_n) d\mu(x_1)...d\mu(x_n),$$

where $n \geq 3$ and $K : \Omega^n \to \mathbb{R} \cup \{\infty\}$. While less common than energies involving a two-input kernel, multivariate energies have appeared in various settings, including geometric measure theory, material science, and discrete geometry.

We discuss current progress in developing theory for multivariate energy optimization, such as semi-definite programming and the consequences of a generalization of positive definiteness. We will also discuss some applications, such as maximizing the squared expected area of a random triangle with vertices on the unit sphere.

The research in this presentation is joint work with Dmitriy Bilyk (University of Minnesota), Damir Ferizović (Graz University of Technology), Alexey Glazyrin (University of Texas Rio Grande Valley), Josiah Park (Texas A
M University), and Oleksandr Vlasiuk (Florida State University).

(Die Abhaltung ist abhängig von der Coronalage - sofern möglich in Präsenz, ansonsten online via Webex)

P. Grabner