Einladung

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Diophantine approximation with Pisot numbers

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Abstract:
Let $X = (x_n)_{n=1}^{\infty}$ be a sequence of real numbers satisfying linear recurrence

$$x_n = a_1 x_{n-1} + a_2 x_{n-2} + \ldots + a_k x_{n-k} + \ldots + a_{d-1} x_{n-d+1} + a_d x_{n-d}.$$

We consider the case when the characteristic polynomial of X is an irreducible polynomial in $\mathbb{Z}[x]$ which is the minimal polynomial of a Pisot number α.
Let

$$L(X) := \sup_{\xi \in \mathbb{R}} \liminf_{n\to\infty} \|\xi x_n\|, \qquad L(\alpha) := \sup_{\xi \in \mathbb{R}} \liminf_{n\to\infty} \|\xi x_n\|.$$

It's easy to see that

$$L(X) = \begin{cases} 0, & \text{if } x_n \to 0 \text{ as } n \to \infty, \\ L(\alpha) > 0, & \text{if } x_n \nrightarrow 0 \text{ as } n \to \infty. \end{cases}$$

The inequality $L(\alpha) > 0$ holds since the sequence X is lacunary when $x_n \nrightarrow 0$ as $n \to \infty$. We call a sequence X lacunary if there exists $\lambda > 1$ such that for all sufficiently large n we have $|x_{n+1}| > \lambda |x_n|$. For any lacunary sequence X A.Y. Khintchin proved that there exist $\xi \in \mathbb{R}$ and $\gamma > 0$ such that for all $n \in \mathbb{N}$ we have $\|\xi x_n\| \geq \gamma$.

Until recently, all known results were just estimates for $L(\alpha)$ if $\alpha \notin \mathbb{Z}$. The first exact value was obtained for $\alpha = \frac{\sqrt{5}+1}{2}$ in [3]. It turned out that in this case $L(\alpha) = \frac{1}{5}$. Then the value of $L(\alpha)$ was calculated explicitly in several other cases ([1],[2],[4]).
Here is a list of the main results.

- If $\sum_{i=1}^{d} a_i$ is odd, then

$$L(\alpha) = \frac{1}{2};$$

- If $\sum_{i=1}^{d} a_i$ is even, then

$$L(\alpha) \le \frac{\sum_{i=1}^{d} |a_i|}{2\sum_{i=1}^{d} |a_i| + 2};$$

- If α is a Pisot number of degree ≤ 3, then $L(\alpha) \ge \frac{1}{5}$;
- If α is a Pisot number of degree ≤ 4, then $L(\alpha) \ge \frac{3}{17}$;
- If α is a Pisot number less than $\frac{\sqrt{5}+1}{2}$, then $L(\alpha) \ge \frac{3}{17}$.

References

[1] Dubickas A. *Distribution of some quadratic linear recurrence sequences modulo 1*, Carpathian Journal of Mathematics 30 (1) (2014), 79-86.
[2] Dubickas A. *On the fractional parts of powers of Pisot numbers of length at most 4*, Journal of Number Theory 144 (2014), 325-339.
[3] Zhuravleva V. *Diophantine approximations with Fibonacci numbers.* J. de Theor. des Nombres de Bordeaux 25 (2013), 499-520.
[4] Zhuravleva V. *Diophantine approximations with Pisot numbers*, preprint at arxiv:1406.0518 (2014).

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