Classification of orbits in $\mathbb{F}^2 \otimes \mathbb{F}^3 \otimes \mathbb{F}^r$, $r \geq 1$

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Tensor products play an important role in both mathematics and physics, with applications in e.g. complexity theory, algebraic statistics, tensor networks in quantum information theory, etc. Most of the research on tensor products deals with vector spaces over algebraically closed fields (complex numbers), and few consider the case where the ground field is finite. In this talk we introduce the relevant problems, give some historical context and report on the classification of the orbits of elements of the tensor product spaces $\mathbb{F}^2 \otimes \mathbb{F}^3 \otimes \mathbb{F}^r$, $r \geq 1$, under the action of two natural groups (the projective stabiliser G of the associated Segre variety and its subgroup H of index two), for all finite; real; and algebraically closed fields. The results are obtained by careful geometric analyses of the different contraction spaces, and their rank distribution.

Theorem 1. *The number of H-orbits of tensors in $\mathbb{F}_q^2 \otimes \mathbb{F}_q^3 \otimes \mathbb{F}_q^r$ is as listed in the following table:*

r	1	2	3	4	5	≥ 6
$\#H\text{−}orbits$	3	10	21	28	30	31

Theorem 2. *The number of G-orbits of tensors in $\mathbb{F}_q^2 \otimes \mathbb{F}_q^3 \otimes \mathbb{F}_q^r$ is as listed in the following table:*

r	1	2	3	≥ 4
$\#G\text{−}orbits$	3	9	18	$\#H\text{−}orbits$

Theorem 3. *If $\mathbb{F}$ is an algebraically closed field or the field of real numbers, then the number of H-orbits and G-orbits of tensors in $\mathbb{F}^2 \otimes \mathbb{F}^3 \otimes \mathbb{F}^r$ is as listed in the following tables.*

r	1	2	3	4	5	≥ 6	
$\#H\text{−}orbits$	3	9	18	24	26	27	$\mathbb{F}$ *algebraically closed*
$\#H\text{−}orbits$	3	10	20	27	29	30	$\mathbb{F} = \mathbb{R}$

r	1	2	3	≥ 4	
$\#G\text{−}orbits$	3	8	15	$\#H\text{−}orbits$	$\mathbb{F}$ *algebraically closed*
$\#G\text{−}orbits$	3	9	17	$\#H\text{−}orbits$	$\mathbb{F} = \mathbb{R}$

[1] M. Lavrauw and J. Sheekey: Classification of subspaces in $\mathbb{F}_q^2 \otimes \mathbb{F}_q^3$ and orbits in $\mathbb{F}_q^2 \otimes \mathbb{F}_q^3 \otimes \mathbb{F}_q^r$. To appear in J. Geom. (arXiv:1503.07894).

[2] M. Lavrauw and J. Sheekey: Canonical forms of $2 \times 3 \times 3$ tensors over the real field, algebraically closed fields, and finite fields. *Linear Algebra Appl.* 476 (2015), 133–147.