Institut für Analysis und Zahlentheorie

Zahlentheoretisches Kolloquium

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Seminarraum Analysis-Zahlentheorie, Kopernikusgasse 24, 2.OG

Diversity in Rationally Parameterized Fields

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Let $F(x, y) \in \mathbb{Q}[x, y]$ be an irreducible polynomial of degree $d > 1$ in y. Hilbert's Irreducibility Theorem (HIT) states that for the vast majority of integers n the polynomial $F(n, y) \in \mathbb{Q}[y]$ is irreducible, i.e. $[\mathbb{Q}(\theta_n) : \mathbb{Q}] = d$ for any root θ_n of $F(n, y)$. However, HIT does not answer the following questions:

> Given an integer N, what is the degree of $\mathbb{Q}(\theta_1, \theta_2, \ldots, \theta_N)$? How many distinct fields are there among $\mathbb{Q}(\theta_j)$, $1 \leq j \leq N$?

These questions were first studied by Dvornicich and Zannier, who showed that there is a positive constant c such that $\mathbb{Q}(\theta_1, \ldots, \theta_N) \geq e^{cN/\log N}$, and consquently that there are at least $c'N/\log N$ many distinct fields among $\mathbb{Q}(\theta_j)$ with $j \leq N$.

We will consider the larger set of fields $\mathbb{Q}(\theta_r)$ where $r \in \mathbb{Q}$ varies over rational numbers of height $H(r) \leq N$. Under some assumptions on F we will obtain a lower bound on the number of distinct fields among $\mathbb{Q}(\theta_r)$, $H(r) \leq N$.

Ch. Elsholtz