Institut für Analysis und Zahlentheorie

Mathematisches Kolloquium

17.01.2020, 14:00 Uhr

Seminarraum Analysis-Zahlentheorie, Kopernikusgasse 24, 2.OG

BOUNDED EXPONENTIAL SUMS

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Let A $\subset \mathbb{N}$, $\alpha \in (0, 1)$, and for x $\in \mathbb{R}$ let e(x) := $\mathrm{e}^{2\pi i x}$. We set

$S_A(\alpha,\mathrm{N}) := \sum$ n$\in$A$n\leq$N e(nα)

Recently, Lambert A'Campo proposed the following question: is there an infinite non-cofinite set A $\subset \mathbb{N}$ such that for all $\alpha \in (0, 1)$ the sum $S_A(\alpha,\mathrm{N})$ has bounded modulus as N $\to +\infty$? In this talk I will give an idea of why such sets do not exist. To show this, I use a theorem by Duffin and Schaeffer on complex power series. The above result can also be extended to prove that if the sum $S_A(\alpha,\mathrm{N})$ is bounded in modulus on an arbitrarily small interval and on the set of rational points, then the set A has to be either finite or cofinite. On the other hand, it can be shown that there are infinite non-cofinite sets A such that $|S_A(\alpha,\mathrm{N})|$ is bounded for all $\alpha \in E \subset (0, 1)$, where E has full Hausdorff dimension and $\mathbb{Q} \cap (0, 1) \subset E$.

R. Tichy