Institut für Analysis und Zahlentheorie

Zahlentheoretisches Kolloquium

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On the maximum of inclomplete Kloosterman sums

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Let $t : \mathbb{F}_p \to \mathbb{C}$ be a complex valued function on $\mathbb{F}_p$. A classical problem in analytic number theory is bounding the maximum

$$M(t) := \max_{0 \leq H < p} \left| \frac{1}{\sqrt{p}} \sum_{0 \leq n < H} t(n) \right|$$

of the absolute value of the incomplete sums $\frac{1}{\sqrt{p}} \sum_{0 \leq n < H} t(n)$. In this very general context one of the most important results is the Pólya-Vinogradov bound

$$M(t) \leq \left\| \hat{t} \right\|_\infty \log 3p,$$

where $\hat{t} : \mathbb{F}_p \to \mathbb{C}$ is the normalized Fourier transform of t. In this talk, we provide a lower bound for certain incomplete Kloosterman sums, namely we prove that there exists a subset of $a \in \mathbb{F}_p^\times$ such that

$$M(e((ax + \overline{x})/p)) \geq \left(\frac{2}{\pi} + o(1) \right) \log \log p,$$

as $p \to \infty$. We prove this by studying the growth of the moments of $\{M(e((ax + \overline{x})/p))\}_{a \in \mathbb{F}_p^\times}$. This is a joint work with Pascal Autissier and Youness Lamzouri.