QMC for unbounded integrands

Classical QMC:

1.) Let $(x_n)_{n\in\mathbb{N}}$ be a low-discrepancy sequence (LDS) in $[0,1)^d$

$$\boxed{D_n^*(x) \leq C\,\frac{\log(n)^d}{n}}$$

2.) Let $f:[0,1)^d \longrightarrow \mathbb{R}$ be a function of bounded variation in the sense of Hardy & Krause.

Then $$\boxed{\left|\int_{[0,1)^d} f(x)\,dx - \frac{1}{N}\sum_{n=1}^{N} f(x_n)\right| \leq V(f)\, C\,\frac{\log(n)^d}{n} \longrightarrow 0}$$

If, instead of 2.), $f : [0,1]^d \to \mathbb{R}$ is
Riemann - integrable, then still

$$\frac{1}{N} \sum_{n=1}^{N} f(x_n) \xrightarrow[N \to \infty]{} \int_{[0,1)^d} f(x)\,dx$$

$\boxed{\text{RATE ARBITRARILY BAD}}$

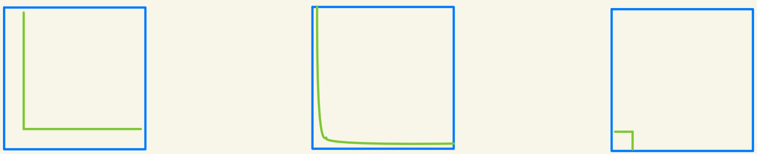
Even if f is very regular, but unbounded,
then anything can happen:

$$\lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} f(x_n) \begin{cases} \bullet \text{ does not exist} \\ \bullet \in \{-\infty, \infty\} \\ \bullet \text{ equals } \int_{[0,1]^d} f(x)\,dx \\ \bullet \text{ equals } \alpha \in \mathbb{R},\ \alpha \neq \int_{[0,1)^d} f(x)\,dx \end{cases}$$

We need more than low discrepancy from the sequence!

Sobol' 1973, Hartinger, Kainhofer, Tichy 2004,
Owen 2004, Hartinger, Kainhofer, Ziegler 2005, ...

consider "corner avoiding property" CAP
resp. properties



Halton-, Sobol'-, Faure-, Niederreiter-
sequences all have CAPs.