The Danzer problem

Daniel El-Baz

TU Graz

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Open problem

Problem (Danzer, 1965)

For $d \geq 2$, does there exist a set in $\mathbb{R}^d$ with finite density which intersects every convex body of volume 1?

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Definition (density)

For $d \geq 2$ and $S \subset \mathbb{R}^d$, define its (upper) density to be

$$\limsup_{R \to \infty} \frac{\#(S \cap B(R))}{R^d},$$

where $B(R) \subset \mathbb{R}^d$ is the ball centred at the origin with radius $R > 0$.

Known results?

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Negative results: unions of lattices (Bambah–Woods, 1971), cut-and-project constructions, vertex sets of aperiodic tilings do not work (Solomon–Weiss, 2016).

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Positive results: there is such a set if one relaxes the condition that $\#(S \cap B(R)) = O(R^d)$ to $\#(S \cap B(R)) = O(R^d \log^{d-1} R)$ (Bambah–Woods, 1971); true already if relaxed to $\#(S \cap B(R)) = O(R^d \log R)$ (Solomon–Weiss, 2016).

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It is possible to come up with many variations on that problem: some are settled, others are still wide open. I invite you to check out the very recent survey by Faustin Adiceam (arXiv:2010.06756).

The end

Thanks for your attention!