# Tales of Our Forefathers

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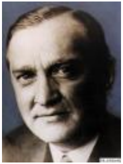
# Introduction

This is not a mathematics talk but it is a talk for mathematicians. It is dedicated to the notion that it would be good that our students realize that while Hahn–Banach refers to two mathematicians,



Mittag-Leffler refers to one.

# Introduction



While on the subject of names, I can't resist a semi-personal story. In the early 1980s, Mike Reed visited the Courant Institute and, at tea, Peter Lax took him over to a student who Lax knew was a big fan of Reed–Simon.

# Introduction



Lax: This is the Reed of Reed–Simon.

Student, mouth falling open: You're Reed.
I thought Reed was Simon's first name.

But I digress—and not for the first time.

# Some Caveats



Four caveats: First, I am not a historian and I've no faith that all that I'm telling you is true. None of the stories was made up, at least by me.

Second, I regret that this is only about forefathers and not foremothers also, although two female mathematicians have cameos later. It is an unfortunate aspect of history that we used to ignore half our mathematical talent—I'm glad we no longer do quite that badly.

# Some Caveats



A third caveat is that I'm an analyst and I learned many of these stories when working on the Notes for a series of Analysis texts that I've written, so I'll be focusing on analysts.

Of course, prior to the twentieth century, mathematicians were more universal and so "analysts" means most mathematicians.

# Some Caveats

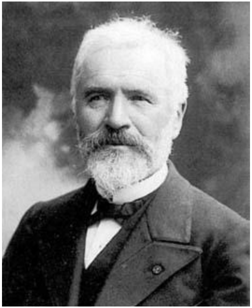
This is true not only of transcendent figures like Leonard Euler (1707–83), Carl Friedrich Gauss (1777–1855), and Bernhard Riemann (1826–66), but also of lesser figures.



# Some Caveats



For example, consider Camille Jordan (1838–1922),



who made significant contributions to algebra (Jordan normal form, Jordan–Hölder sequences), geometry (Jordan curves), and analysis (Jordan content and Jordan decomposition of functions of bounded variation).

# Some Caveats



By the way, this is not the Jordan of Jordan algebras and the Jordan–von Neumann theorem—that was the physicist Pascual Jordan (1902–80), best known as one of the authors of the "three-man paper," which was one of the foundational papers of quantum mechanics.

# Some Caveats



The other two men were Heisenberg and Born, both of whom got Nobel prizes for their contributions to quantum theory (1932 and 1954, respectively).



Some have speculated that <u>this</u> Jordan might have shared in Born's Nobel prize if it weren't for his strong support of the Nazis and pro-Nazi views during the Hitler era.

# Some Caveats

A last caveat: Mostly we remember mathematicians by applying their names to theorems and to mathematical objects. In this regard, I quote two principles which appeared in a 1997 lecture of V.I. Arnold (which he claims were formulated by M. Berry):

**The Arnold Principle.** "If a notion bears a personal name, then this name is not the name of the discoverer."

# Some Caveats

**The Berry Principle.** "The Arnold Principle is applicable to itself."

Berry's Principle is certainly true. You won't find "Arnold's Principle" on Wikipedia, but you will find "Stigler's law of eponomy," which Stigler stated in 1980 as

# Some Caveats



**Stigler's Law.** "No scientific discovery is named after its original discoverer."

In fact, Stigler said that this principle was due to Merton (not the Nobel Prize winner in economics for work in financial math, but his father, a distinguished sociologist).

Stigler remarked that since it was a discovery of Merton, it was appropriate to name it Stigler's law to validate the law!

# Family Matters



One thing we lose sight of when we just think of mathematicians as names on theorems is that mathematicians are people with parents, children, wives, and in-laws that impact their lives. Particularly interesting are mathematicians with familial relations: father/son, brothers, father-in-law/son-in-law.

We start with the largest of the mathematical families.

# The Bernoullis

The family originally fled Belgium for religious reasons and wound up in Basel some time before the birth of mathematicians. The senior mathematician was Jacob (1654–1705).


There were his younger brother Johann

and Johann's son Daniel (1700–82)

who was born in Holland where his father was teaching (at the time). There were also several lesser cousins.

# The Bernoullis



Jacob was the most significant mathematically with the discovery of $e$ as the limit of $(1 + n^{-1})^n$, Bernoulli trials and the law of large numbers, and Bernoulli numbers. Much of his most famous work appeared posthumously (1713) in *Ars Conjectandi*.

Daniel is most noted for Bernoulli's principle in hydrodynamics and Johann for contributions to differential equations, for early work in the calculus of variations, and, as we'll see, for l'Hôpital's rule.

# The Bernoullis

To use modern parlance, the family was dysfunctional. Johann and Jacob were two in a family of ten and Jacob was Johann's senior by twelve years.

There was tremendous jealousy between the two brothers. Once Johann got recognition, Jacob declared Johann his student and Johann objected. There was a huge priority fight between them over the isoperimetric problem and a total break in 1697. Jacob was convinced that Johann wanted his chair in Basel, which Johann got after Jacob's death.

# The Bernoullis

You may have noted the eight years between Jacob's death and the publication of his posthumous work—this was due to squabbling between Johann and nephew Nicholas.

The most shocking event involved books on hydrodynamics. In 1738, Daniel published a book on the subject that he had largely finished in 1734. His father then published a book on the same subject using many of Daniel's ideas, predated his book claiming it was earlier and that Daniel had taken the ideas from him!

# The Bernoullis

Guillaume François Antoine, Marquis de l'Hôpital (1661–1704) was a French nobleman who over many years paid Johann Bernoulli a large annual retainer, initially for lectures on the new calculus of Leibnitz and Newton and for continuing advice.

# The Bernoullis

In 1696, l'Hôpital published "Analyse des Infiniment Petits pour l'Intelligence des Lignes Courbes," a hit as the first textbook on differential calculus. It contained what has come to be called l'Hôpital's rule.

He thanked various people in the Preface, including Johann Bernoulli. But after l'Hôpital's death, Bernoulli claimed that the book was close to a verbatim copy of the notes of the lectures he gave to l'Hôpital.

Ironically, given Johann's other priority disputes, this claim was dismissed by historians of mathematics of the nineteenth century, until in the 1920s when notes were found in the University of Basel which supported Johann's claim!

# Euler



Undoubtedly, Johann Bernoulli's greatest contribution to mathematics concerns Leonhard Euler (1707–83).

Both Euler's father and maternal grandfather were pastors who expected him to go into the family business. During his studies, Euler convinced Johann to give him private lessons. Johann had been a student in university with Euler's father and was able to convince the father to allow Euler to go into mathematics rather than become a pastor.

# Euler

Euler, who also was a great physicist, was very prolific. It is estimated that about one-third of all research papers in mathematics and physics in the eighteenth century were written by him. In 1775, at age 68, he wrote over fifty papers. The remarkable thing about that is that he had been totally blind since 1766! He wrote with the help of scribes and mathematical assistants (among them his son and grandson).

# Euler

He was also the author of textbooks on calculus, mechanics, and many other areas—some in use for a hundred years—and the first great popular book on science. His complete works (not counting the volumes of correspondence) run to 72 volumes, each in the 400–700 page range!

# Weierstrass

Karl Weierstrass (1815–97) was the son of a Prussian finance ministry bureaucrat who wanted his son to follow in his footsteps and forced him to study finance at the University of Bonn.
Karl rebelled and quit just short of his degree.
After negotiations by a friend of his father, the compromise reached was that Karl could get a degree from Münster that would allow him to teach mathematics in gymnasium.

# Weierstrass

He taught at gymnasium starting in 1841, and during the 1840s wrote unpublished works that established the Weierstrass approach to complex analysis centered on power series. Many were only published in his complete works fifty years later although, to get ahead in our story, he exposed many of them in his lectures at the University of Berlin.

During the summer of 1853, he wrote a mémoire on elliptic functions. In the hands of Abel and Jacobi, the subject had reached maturity around 1830, so the solution of the Jacobi inversion problem for general hyperelliptic functions caused a sensation.

# Weierstrass



Shortly afterwards, Weierstrass was a university professor and then one at the University of Berlin where he developed an active school of analysis. His lack of a degree was settled by arranging for him to get an honorary degree.

# Weierstrass



One of my favorite quotes about Weierstrass is from T.W. Körner's book on Fourier analysis, commenting on Fejér's theorem on Cesàro summability of Fourier series and Weierstrass' theorem on density of polynomials in $C([0,1])$:

*"Fejér discovered his theorem at the age of 19, Weierstrass published this theorem at the age of 70. With time, the reader may come to appreciate why so many mathematicians regard the second circumstance as even more romantic and heart warming than the first."*

# Fejér

Lipót Fejér (1880–1959) was born Lipót Weiss (German for white) in Hungary and was a student of Hermann Schwarz (German for black). In high school, he changed his name to Fejér (Hungarian for white), in part because he expected less anti-Semitism. One of his students was Fekete (Hungarian for black!). His other students included: M. Riesz, Pólya, Szegő, von Neumann, Turán, and Erdös.






# Stone

While on the subjects of fathers and polynomial approximation, I note that the father of Marshall Stone (1903–1989) was Harlan Stone (1872–1946), who was a chief justice of the U.S. Supreme Court.

# Bastards

The parents of Stefan Banach (1892–1945) were not only not married, but his mother departed four days after his birth, leaving behind nothing but the name Banach. Stefan was raised initially by his paternal grandparents and then by friends of his father. As a teenager, he was left to fend on his own. While he did study some mathematics, he only managed a first degree in Engineering.

# Banach

His life changed dramatically because of the following incident in 1916 when Banach was 24. Hugo Steinhaus (1887–1972), five years Banach's senior, had been appointed a professor in Lwów, but waiting for the position to start, he was living in Kraków and made a habit of strolling in the evening. He heard wafting through the air "Lebesgue integral" and followed the voices to find Banachand his friend Otto Nikodym (1887–1974).

# Banach

Steinhaus, who regarded Banach as his greatest mathematical discovery, took Banach to Lwów where, first, Banach got a graduate degree (his dissertation defined and began the study of what we now call Banach spaces) and then, with Steinhaus, founded the famous Lwów school and the journal *Studia Mathematica*.

# Steinhaus Stories



Steinhaus was a rather pleasant person. I was told by Mark Kac, one of his students, that Steinhaus loved stories and bon mots. A favorite among the ones Kac passed on was:

*"The acceptance of your work by the mathematical public goes through three phases:*

First, they say it's wrong.

Then, they say it's trivial.

Finally, they say I did it first."

# d'Alembert

Jean-Baptiste le Rond d'Alembert (1717-1783) was found abandoned in the church of Saint-Jean-le Rond in Paris named after John the Baptist. He had been abandoned by his mother, Claudine Guérin de Tencin, whose literary salon was a social center during the reign of Louis XV. Her many lovers included Richelieu and Louis-Camus Destouches, an army officer who was d'Alembert's father.

# d'Alembert

While neither parent officially acknowledged d'Alembert, his father did arrange a foster home where d'Alembert lived for almost fifty years and, when he died, d'Alembert was left an income that allowed him to pursue mathematics rather than the more mundane law that he'd studied.

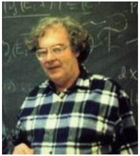
d'Alembert discovered the wave equation as describing plucked strings and found the general one dimensional solution. He was an editor with Diderot of the *Encyclopédie* which led to his being made a member of Académie Française (the immortals). Laplace was his student.

# Kolmogorov

Andrei Kolmogorov (1903–87) was also a bastard. His mother died in childbirth and his father had nothing to do with him. He was raised by his mother's sister; Kolmogorov was his maternal grandfather's name. In Soviet Russia, he was able to get an education. He was also a wunderkind. Just as Fejér made a great contribution to Fourier series at age 19, Kolmogorov was 19 when he found $L^1$ functions on the unit circle whose Fourier series is a.e. divergent.

# Kolmogorov

He, of course, went on to make numerous high-order contributions to probability theory and to dynamics as well as to topology and computer science. His students include Arnol'd, Dobrushin, Dynkin, Gel'fand, and Sinai.


# Luzin

Kolmogorov was an important player in the Luzin affair of the 1930s. His teacher was Nikolai Luzin (1883–1950), in turn a student of Dmitri Egorov (1869–1931). Both were victims of the 1930s Stalin reign of terror.

Egorov was religious and loudly objected to the Soviet treatment of his beloved Russian church.He was dismissed from his post in 1929, arrested in 1930, and died in the middle of a hunger strike in 1931.

# Luzin

Luzin was the center of a lively group of younger mathematicians in Moscow in the 1920s. Included in what was called Luzitania were his students Alexandrov, Khinchine, Kolmogorov, Souslin, and Urysohn. He was a powerful figure in the Russian Academy.


In 1936, Luzin was accused of anti-Soviet behavior and given what was essentially a show trial before a commission of the Academy. He was found guilty but received a mild "sentence"—basically, a loss of power and influence that left him a broken man.

# Luzin



Among those testifying against Luzin were Alexandrov, Khinchine, and Kolmogorov. I think of this as a kind of mathematical patricide—and it has elements of Greek tragedy.

There has been widespread speculation about the motivation of Alexandrov and Kolmogorov. These two were very close—they traveled together and shared a house. Whether they were having a homosexual relationship or only gave the appearance of one, there is a belief that they were pressured by the KGB to testify against Luzin or be arrested for homosexual behavior.

On January 17, 2012, the Russian Academy formally rescinded their motion condemning Luzin.

# Littlewood



Of course, the production of a bastard involves two people besides the innocent child! In this regard, consider Sir John Littlewood.

Just as one can't imagine discussing Stan Laurel without Oliver Hardy, one can't imagine John Littlewood (1885–1977) without G.H. Hardy (1877–1947).

They are arguably the most celebrated and most successful mathematical collaboration ever.

# Littlewood

At one point, Harald Bohr

(a distinguished mathematician; in the early years, more famous than his brother Niels because Harald was on Denmark's Olympic medal-winning soccer team), said:

"These days there are three great British mathematicians: Hardy, Littlewood, and Hardy–Littlewood."

# Littlewood

Both Hardy and Littlewood were bachelor dons. With nice rooms and meals (the famous high table), there was a tremendous bonus to staying single. Littlewood spent his entire career (except for a brief postdoc in Manchester) in Cambridge. Hardy also started there but was unhappy during the First World War due to Russell's dismissal (for pacificism) and because he was not virulently anti-German. He left for Oxford in 1919, but despite preferring Oxford, returned to Cambridge in 1931 because in Cambridge, unlike Oxford, one could keep one's rooms after retirement.

Despite being a bachelor, in his later years, Littlewood traveled with his "niece," Ann, who he eventually acknowledged was his daughter.

# Littlewood



I have a favorite (perhaps apocryphal) story about Littlewood. Until his later years when he was helped by medication, Littlewood suffered from severe depression, so much so that he didn't travel. Hardy, however, was an inveterate traveler, widely known personally outside England.There is a story involving a visit to England by (in some versions) Landau and (in some) by Wiener,

who upon meeting Littlewood exclaimed: "Oh, you exist. I thought Littlewood was a pseudonym Hardy used for his weaker papers."

# Littlewood

As for Hardy's view, in his autobiography, *A Mathematician's Apology*, he said the following which was borrowed for the recent movie, *The Man Who Knew Infinity*:

I still say to myself when I am depressed and find myself forced to listen to pompous and tiresome people: *Well, I have done one thing you could never have done, and that is to have collaborated with Littlewood and Ramanujan on something like equal terms.*

# Fathers-in-Law

Kummer (1810–93) was Schwarz's father-in-law. Hermite (1822–1901) was Picard's (1856–1941). Here there is an interesting timeline. Picard got his degree, went off to Toulouse, proved his famous theorems, and only then, returned to Paris and married Hermite's daughter.

Laurent Schwartz (1915–2002), of whom more soon, was Paul Lévy's (1886–1971) son-in-law and John Tate (1925–) is Emil Artin's (1898–1962). Richard Courant (1888–1972) who was the father-in-law of both Jürgen Moser (1928–1999) and, (long after Courant's death) of Peter Lax (1926-..., whose second wife is the younger sister of Moser's widow) was in turn the son-in-law of Carl Runge (1856–1927) who in turn married the niece of Paul du Bois-Reymond (1831–1889).

# Fathers-in-Law

Schwartz's great uncle was Jacques Hadamard (1865–1963), but it appears he had no mathematical influence other than to express dismay that the 16-year old budding mathematician did not know about the Riemann zeta function! Alfred Pringsheim (1850–1941) was the father-in-law of Thomas Mann. The biologist Paul Erhlich (1854–1915) was the father-in-law of Edmund Landau (1877–1938) who was, in turn, the father-in-law of Isaac Schoenberg (1903–1990). When he was 30, Émile Borel (1871-1956) married the 17 year old daughter of the mathematician Paul Appell (1855-1930). Besides his mathematics, Borel was French Minister of the Navy from 1925-1940. After World War II, he received the Grande Croix Légion d'Honneur for resistence activities (he was 68 when the war began).

# Fathers-in-Law

Here's a story of a non-mathematical father-in-law. George Airy (1801–92) came from a poor background but managed to get through Cambridge by being a sizar (part-time man servant!). In 1824, he met and fell in love with Richarda Smith, the daughter of the vicar of Chatsworth. In 1826, at age only 25, he was appointed the Lucasian Chair of Mathematics. This is the most prestigious chair in science. Among its other holders are Newton, Dirac, and Hawking.

# Fathers-in-Law

But Vicar Smith would not allow Airy to marry Richarda because the Lucasian chair only paid $\pounds 100$ per year. In 1830, the Plumian Chair of Astronomy, which paid $\pounds 500$, opened, Airy got it and Richarda! He went on to become Astronomer Royal.

# Grace Chisholm Young



Grace Chisholm (1868–1944) studied mathematics in Cambridge with William Young (1863–1942), her tutor.

She then went to Göttingen and got a degree in 1895 supervised by Felix Klein.

She returned to England, married Young whom she encouraged to become active in research (he had not been!).

# Grace Chisholm Young

Works credited to him include the independent rediscovery of Lebesgue integration two years after Lebesgue, the Hausdorff–Young inequality, Young's convolution inequality, and Young's inequality on conjugate convex functions (e.g., $xy \leq \frac{x^p}{p} + \frac{y^q}{q}$). He is not the Young of Young tableaux—that is Alfred Young (1873–1940), a clergymen and amateur mathematician.

# Grace Chisholm Young



It is clear that some of Young's work was joint work with Grace, but not clear which. He wrote to his wife at one point:

*"The fact is that our papers ought to be published under our joint names, but if this were done neither of us would get the benefit of it. No. Mine the laurels now and the knowledge. Yours the knowledge only. Everything under my name now, and later when the loaves and fishes are no more procurable in that way, everything or much under your name. At present you cannot undertake a public career. You have your children."*

# Weyl and Schrödinger



Erwin Schrödinger (1887–1961)

and Hermann Weyl (1885–1955)

were both professors in Zurich in the 1920s, coupled scientifically in work on quantum mechanics.
But they were linked not only scientifically.

# Weyl and Schrödinger

As one biographer of Schrödinger put it:

*"Those familiar with the serious and portly figure of Weyl at Princeton would have hardly recognized the slim, handsome young man of the twenties, with his romantic black moustache. His wife, Helene Joseph, from a Jewish background, was a philosopher and literateuse. Her friends called her Hella, and a certain daring and insouciance made her the unquestioned leader of the social set comprising the scientists and their wives.*

# Weyl and Schrödinger



*Anny, Schrödinger's wife, was almost an exact opposite of the stylish and intellectual Hella, but perhaps for that reason Weyl found her interesting and before long she was madly in love with him . . . The special circle in which they lived in Zurich had enjoyed the sexual revolution a generation before the United States. Extramarital affairs were not only condoned, they were expected, and they seemed to occasion little anxiety. Anny would find in Hermann Weyl a lover to whom she was devoted body and soul, while Weyl's wife Hella was infatuated with Paul Scherrer."*

# Lyapunov

Introduction
Family Matters
Educational Follies
Underappreciated Mathematics
Death of a Mathematician

We close our tales of families with a love story. Alexander Mikhailovich Lyapunov (1857–1918) is known for his work on stability of ODEs (Lyapunov exponents) and on the Central Limit Theorem.


In 1886, he married Natalia Sechenov—he'd met her as a teenager when he was being tutored by her father, his cousin.

# Lyapunov

Lyapunov was a student of Chebyshev in St. Petersburg. He took at position in Kharkov in 1884 and, with Chebyshev's death, returned to the latter's position in St. Petersburg.

In 1917, Lyapunov took a position in Odessa since the doctors thought the climate there was better for Natalia's tuberculosis. Her condition worsened and she passed away on October 31, 1918. Later that day, the distraught Lyapunov shot himself, dying of his wounds three days later.

# Dirichlet



We now turn to issues involving education and degrees. Maybe it will help you deal with bureaucratic absurdities and educational setback to know the greats were also.

Johann Peter Gustav Lejeune Dirichlet (1805–59) was a German of Belgian extraction ("the young one from Richlet"). Because of the poor education in Germany at the time, he went to Paris and studied with Fourier and Poisson. After his famous work on Fourier series, the Germans wanted him to return to a professorship, but the lack of a German degree was a problem. Like Weierstrass later, this was solved by arranging an honorary degree for him.

By the way, Dirichlet's wife was Felix Mendelssohn's sister, and the Dirichlets, Mendelssohns, and Jacobi had close social connections.

# Bernstein

Sergei Bernstein (1880–1968) was a Jewish Ukrainian mathematician known for his work in approximation theory (Bernstein polynomials and inequality) and for his integral representation theorem for completely positive functions.

# Bernstein



He went to study in Paris, spent three years in Göttingen with Hilbert, and then submitted a thesis in Paris which he defended in 1904 before a committee of Poincaré, Hadamard, and Picard.

This thesis solved (or partially solved; later contributions include E. Hopf, de Giorgi, and Nash) Hilbert's Nineteenth Problem—one of the first to be solved.

# Bernstein

Bernstein returned to Russia where his Ph.D. wasn't recognized, so before he could teach, he had to be a graduate student and then submit a master's thesis. Eventually, after he was teaching, he submitted a Ph.D. thesis that went a ways towards solving Hilbert's Twentieth Problem.

# Jensen



In June 2012, I attended a conference in Copenhagen that met in the Danish Academy of Sciences. There was a huge painting of a meeting held around 1900, and a number of famous Danish scientists of the period were pointed out to us.

# Jensen

I asked where was Johan Ludwig Jensen (1859–1925) who revolutionized the study of convex functions (think Jensen's inequality) and invented Jensen's formula—a cornerstone of Nevanlinna theory. I was informed that he spent his career as an engineer for the telephone company (I knew this), didn't have an advanced degree, and so wasn't ever elected to the Academy.

# Cotlar

Mischa Cotlar (1912–2007) is best known for Cotlar's Lemma (aka Cotlar–Knapp–Stein Lemma) which is critical for estimating integral and pseduo–differential operators. He invented it to prove $L^2$ bounds on higher dimensional Hilbert transforms.

Here is what Charlie Fefferman said about his interaction with Cotlar when Fefferman was a postdoc and Cotlar almost 60: "Cotlar was one of the kindest and most self-effacing people I ever met. One of my secret ambitions during the few months we were together in Chicago, long ago, was to succeed in walking through a doorway after Mischa, rather than letting him hold the door for me. In this I failed".

# Cotlar

When Cotlar was born in the Ukraine in 1912, his father was the manager of a local mill. So after the revolution, he was regarded as a pariah and, as punishment, his children were not allowed to attend school, so that Mischa had exactly one formal year of schooling. But his father taught him some mathematics and to play the piano.

In 1928, the family was able to emigrate to Montevideo, Uruguay. The four of them lived in single room; his father sold newspapers on a downtown street corner, his older brother became a tramway conductor and Mischa played the piano from 4pm to 4am in a rough harbor bar! Not too long after arriving, the senior Cotlar came in first in a national chess tournament and the fact that a member of the "lower classes" did this caused some news coverage.

# Cotlar



The articles mentioned that the winner claimed to have a mathematically gifted son. This was noticed by Rafael Laguardia (1906–1980), a professor of mathematics and former student of Picard. He invited Mischa initially to attend his seminar and later to teach a course in number theory. He also helped Mischa upgrade his job to playing in a chamber music trio in the fanciest hotel in Punta del Este, the seaside resort near Montevideo.

In 1934, Professor Julio Rey Pastor (1888–1962), who had been a student of Felix Klein and was alternating his time between Madrid and Buenos Aires, visited Montevideo. Cotlar was drawn to Buenos Aires where he learned more mathematics and shifted from earning his living as a pianist to being a tutor of mathematics. He got a job in a University research institute, but was fired after six months when the authorities learned he had no degree.

# Cotlar

He corresponded with Fréchet who helped him publish a paper and in 1938, he married Yanny Frenkel, a student of Rey Pastor and his wife until his death 70 years later.

From the mid ‘40’s onwards, several American foundations sponsored trips by US mathematicians to Argentina including Adian Albert (1905–1972), George Birkhoff (1884–1944), Marshall Stone (1903–1989) and Antoni Zygmund (1900–1992). During one of these trips Zygmund discovered Alberto Calderón (1920-1998), an engineer and brought him to Chicago with money from the Rockefeller Foundation, where Stone convinced them that Calderón should get a degree.

# Cotlar

Earlier, Birkhoff had met Cotlar and had been so impressed by him that he recommended Mischa for a graduate Guggenheim Fellowship, which they awarded. Alas, Birkhoff died shortly after he received the letter from the Guggenheim Foundation and before he had let Cotlar know, so the letter was lost among his papers until rediscovered by Garret Birkhoff (1911–1996), his son and a noted mathematician in his own right. Even though the initial award was in 1944, it was only in 1950 that Cotlar made it to Yale where he worked with Shizuo Kakutani (1911–2004) completing a PhD. thesis on ergodic theory. But when the Yale adminstration learned he had no prior degrees, they refused to award Cotlar a degree!

# Cotlar

Marshall Stone then stepped in and suggested Mischa come to Chicago, where Stone assured him the administration would be more flexible. Cotlar insisted on doing a new thesis which he wrote on harmonic analysis under Calerón.

Cotlar spent much of his career as a Professor a Professor in Buenos Aires where he helped nurture the talents of Argentine born Carlos Berenstein and Norberto Kerzman. After the 1966 coup by the military junta which ordered beating of students and faculty at the University, Cotlar went into exile Caracas, returning to Buenos Aires after the return of democracy to Argentina.

# High School Teachers

We've already seen that Weierstrass was a high school teacher for many years. Of course, he didn't have an advanced degree. There are many examples of those who got advanced degrees and went on teach high school because that was the best job they could find!

Some did their best work then, often in the evening.

# High School Teachers



Among them are Eduard Kummer (1810–93) [who had Leopold Kronecker (1823–91) as a student in class] and Jacques Hadamard (1865–1963) [who had the twelve-year old Maurice Fréchet (1878–1973) in his class and was later his thesis advisor!]. Also, Henri Lebesgue (1875–1941), who invented his measure during that period, René-Louis Baire (1874–1932) who invented monotone classes of functions, and Rolf Nevanlinna (1895–1980) who developed his value distribution theory while a high school teacher.

# Nevanlinna

His theory of 1925 made a big splash and he was shortly afterwards a professor at the University of Helsinki where he became rector in 1941. He cooperated with the Nazis and, indeed, was the chair of the support committee for the Finnish branch of the Waffen SS.

# Nevanlinna

After the war, he was dismissed as rector and spent some time in Zurich. His defenders claim he was not so much pro-German as anti-Russian. He fought in a Finnish–Russian war at the time just after the Russian revolution, and morally supported Finland in its 1939 war with Russia. He was later rehabilitated enough to be President of the ICM in 1959. Still, I am among those who questioned the appropriateness of an ICM prize in his honor, something recently changed.

# Fourier



Some mathematicians are either unappreciated in their lifetimes or even now. Let me talk about a few.

Joseph Fourier (1768–1830) was more an engineer and physicist than mathematician. Because of his practical abilities, he had high political appointments. He went to Egypt with Napoleon's 1798–99 campaign and ended up governor of Lower Egypt. He spent many years as Prefect under Napoleon of the province that includes Grenoble and constructed the Turin–Grenoble highway. He wound up with an appointment as a Baron.

# Fourier

Fourier is, of course, best known for his book on heat flow, which includes both Fourier series and transforms. He submitted it to the French Academy in 1807.


A committee of Lagrange, Laplace, Monge, and LaCroix questioned the notion of expanding "any" function in Fourier series and it was only in 1822 that the book was published.

In 1829, Fourier's student Dirichlet proved piecewise $C^1$ functions have convergent Fourier series (using the Dirichlet kernel).

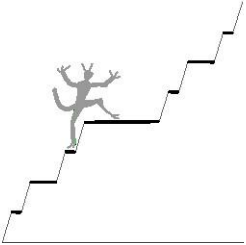
# Fourier

Fourier also studied Egyptian history while there. Here is what Körner says:

*"Apart from his prefectorial duties Fourier helped organise the 'Description of Egypt' . . . Fourier's main contribution was the general introduction–a survey of Egyptian history up to modern times. An Egyptologist with whom I discussed this described the introduction as a masterpiece and a turning point in the subject. He was surprised to hear that Fourier also had a reputation as a mathematician."*

# Cantor

One naively thinks theorems are either true or not and can't be controversial. But radically new approaches can face strong attacks. I want to consider three now-central pillars of modern mathematics, but they were not always so. First, Georg Cantor (1845–1918).



# Cantor



His work on counting infinities—especially his proof of the existence of transcendentals and that $\mathbb{R}$ and $\mathbb{R}^2$ had the same number of points—caused great discomfort. Kronecker was an implacable foe who blocked Cantor's dream of a professorship at Berlin. Poincaré thought it a disease that he hoped would be cured!

# Cantor

Cantor was born in St. Petersburg, spent most of his life in Germany, and has been described variously as Russian, German, Danish, and Jewish. His paternal grandparents were Danish Jews, but unobservant enough that his grandfather gave all his children names of Christian saints. Cantor's father was a Lutheran and his mother was raised as a Russian Roman Catholic. Despite this, Cantor suffered from anti-Jewish prejudice during parts of his career.

His initial burst of activity started in 1873 and ended about 1885, with his discovery of the Cantor function. There was a second few years around 1891. In between and in the later years of his life, Cantor was incapacitated by depression, now believed to be caused by bipolar disorder.

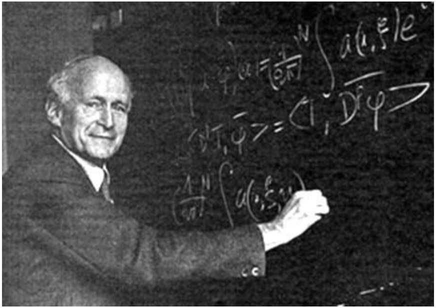
# Lebesgue and Schwartz



Henri Lebesgue and Laurent Schwartz revolutionized analysis. As Strichartz says in his analysis text:

*"Distribution theory was one of the two great revolutions in mathematical analysis in the 20th century. It can be thought of as the completion of differential calculus, just as the other great revolution, measure theory (or Lebesgue integration theory), can be thought of as the completion of integral calculus. There are many parallels between the two revolutions. Both were created by young, highly individualistic French mathematicians (Henri Lebesgue and Laurent Schwartz). Both were rapidly assimilated by the mathematical community, and opened up new worlds of mathematical development. Both forced a complete rethinking of all mathematical analysis that had come before, and basically altered the nature of the questions that mathematical analysts asked."*

# Lebesgue and Schwartz



But the assimilation, while “rapid,” wasn’t overnight. Hermite initially dismissed Lebesgue’s work as insignificant. As for Schwartz, Treves (Schwartz’s student) tells the following story in his obituary for Schwartz:

# Lebesgue and Schwartz



*"In 1948 Laurent Schwartz visited Sweden to present his distributions to the local mathematicians. He had the opportunity of conversing with Marcel Riesz. Having written on the blackboard the integration-by-parts formula to explain the idea of a weak derivative, he was interrupted by Riesz saying, 'I hope you have found something else in your life.'*

"*Later Schwartz told Riesz of his hopes that the following theorem would eventually be proved: every linear partial differential equation with constant coefficients has a fundamental solution (a concept made precise and general by distribution theory). 'Madness!' exclaimed Riesz. 'This is a project for the twenty first century!' The general theorem was proved by Ehrenpreis and Malgrange in 1952."*

# Lebesgue and Schwartz

Part of the irony is that Riesz's students Gårding and Hörmander used distributions to reformulate and study quantum field theory and PDEs, respectively.

# Helly



Eduard Helly (1884–1943) is underappreciated and a paradigm for "may you live in interesting times" being a curse. In 1912, while teaching high school, he wrote a brilliant paper about $C([0,1])$. He proved the Hahn–Banach theorem for this case (they did their work ten years later) using an argument that works for general separable Banach spaces (which had not yet been defined!). He also proved sequential weak compactness of the unit ball in the measures (Alaoglu's work was 25 years later).

# Helly

So why isn't he better known?

When war broke out in 1914, Helly enlisted, serving as a lieutenant. In 1915, he was shot in a lung and was captured by the Russians. He spent the next few years in a hospital and prison in Siberia. He suffered from the lung injury and strain on his heart for the rest of his life. Even after Russian involvement in the World War ended, he was not repatriated because the civil war in Russia made travel difficult. He only returned to Vienna in 1920. While he was able to get a habilitation, he couldn't get a paid academic position — his wife believed this was because he was Jewish and because Hahn, who was himself Jewish, favored someone younger than a person fifteen years past doctorate.

# Helly



Helly found a position in a bank which failed in 1929, but eventually found work as an actuary. After the Nazi occupation of Austria in 1938, Helly and his wife fled to the U.S. There things improved somewhat—with Einstein's aid, he obtained a teaching position in a community college and then a position writing mathematical training manuals for the Signal Corps at the start of America's involvement in World War II. Finally, he was offered a mathematics professorship at Illinois Institute of Technology but died of a heart attack shortly thereafter.

# Fréchet

There are those who think that Maurice Fréchet (1878–1973) was a pivotal figure in twentieth century mathematics. Angus Taylor, who spent his career at UCLA (after getting a Ph.D. in 1936 from Caltech), is among them, and I am sympathetic to this point of view.

Why? In his 1906 thesis, Fréchet defined metric spaces. He didn't have the triangle inequality but a number of alternatives that included it (shortly afterwards, F. Riesz focused on the triangle inequality).

# Fréchet



Why was this work so important? Before Fréchet, analysts studied individual functions or subsets of $\mathbb{R}^n$. Hadamard (his advisor) and some Italian analysts considered $C([0, 1])$ as a space, but that's as far as it went. Fréchet defined his metrics on an arbitrary set. Abstraction, which has been so successful in modern mathematics (not just in analysis), had its roots in Fréchet's work.

# Fréchet

So why hasn't he gotten credit? For one thing, his initially revolutionary idea has become so accepted that we cannot recall it wasn't always there. Secondly, his contributions are conceptual rather than solving some long-open problem or proving some big theorem (although he did find the dual of an inner product space independently of F. Riesz). Finally, his contributions to the foundations of topology were downplayed by Bourbaki relative to Hausdorff's 1914 book.

I end my discussion of Fréchet with dueling quotes: the first is from Dieudonné's comment to Taylor about the naming of Fréchet space:

# Fréchet

*"Fréchet was always striving for generality without caring for applications, and this was thoroughly repugnant to the Bourbaki spirit, where no notion could be accepted if we could not be convinced that it was useful in some classical problem (although many readers, for lack of background, did not realize it). Nevertheless, we thought that Fréchet's name deserved to be attached to those spaces, not so much for his 1926 paper, but because in his 1906 thesis."*

The second is from a letter the 71-year old Alexandrov sent to "Cher Maitre et ami," the 89-year old Fréchet (my translation):

*"What is your place and role—it is a place among the greatest mathematicians of our time, it is the role of a true master."*

# Prime Number Theorem

We end with death. So it isn't such a downer, I begin by noting that many mathematicians have lived to ripe old ages—70s, 80s, 90s, and even over 100! Indeed, as Odlyzko, as quoted in Derbyshire's "Prime Obsession," said:

*"It was said that whoever proved the Prime Number Theorem would attain immortality. Sure enough, both Hadamard and de la Vallée Poussin lived into their late nineties. It may be that there is a corollary here. It may be that the Riemann Hypothesis is false: but, should anyone manage to actually prove its falsehood—to find a zero off the critical line—he will be struck dead on the spot, and his result will never become known."*

# Prime Number Theorem



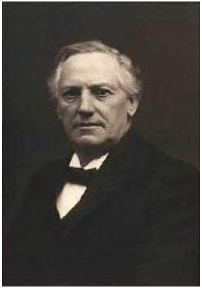
Indeed, Hadamard lived to 97 (but saw sons die in both World Wars) and de la Vallée Poussin to 95.
Erdös and Selberg, who found the first "elementary" proofs, lived to 83 and 90, respectively.




The irony of the quote is that Odlyzko has done computer searches to find zeros off the critical line.

# Accidental Death

As we know from Schramm's death, accidental death is still with us. Perhaps the strangest accidental death of a mathematician was Jorgen Gram (1850–1916) of Gram–Schmidt. He was walking to an Academy meeting when he was struck by a bicycle and killed.I think of Gram when watching bicycles whizzing by in Copenhagen.



# Urysohn

Pavel Urysohn (1898–1924) was noted for his proof that any second countable, normal topological space is metrizable—during which he used what has been called Urysohn's lemma. In 1924, he and his friend Alexandrov traveled to Göttingen and Paris and on to vacation in Brest on the coast of France.While swimming, he was swept off by a wave and perished at age 26.

# Paley

Raymond Paley (1907–33) did remarkable research in harmonic analysis with Littlewood and with Pólya and with Zygmund in Cambridge. He went to the U.S. to work with Wiener, and there went on a skiing vacation in Banff where he was killed in an avalanche, also at age 26.

# Medical Limitations

Modern medicine can be best appreciated by thinking about deaths in the nineteenth and early twentieth centuries that would likely have been avoided with current technology.

Niels Abel (1802–29),Gotthold Eisenstein (1823–52),and Bernhard Riemann (1826–66) died of lung ailments at ages 26, 29, and 39. Thomas Jan Stieltjes (1856–94) at age 38, although I have been unable to find out what the cause was, other than an illness. Hermann Minkowski (1864–1909) died at age 44 of a burst appendix.


# Riemann

What is perhaps most amazing about Riemann is that he has only about a dozen papers, several of them posthumous. There are six monumental multifaceted masterpieces. One had the Riemann integral as a lead-in to Fourier series and the Riemann–Lebesgue lemma, and Riemann local convergence theorem. Another had his "basic" complex analysis: Cauchy–Riemann equations, Riemann removable singularities, Riemann mapping theorem, and Riemann surfaces of functions. A single paper has all of Riemann geometry: from metric to geodesics to Riemann curvature.

# Riemann

There is the celebrated short paper on the Riemann zeta function, its functional equation, the Riemann hypothesis, and his vision of the complex analytic view of the distribution of primes. And there are papers on higher-dimensional theta functions (and Riemann–Roch) and on the Riemann approach to hypergeometric functions (and monodromy).

# Minkowski

Hilbert's tribute is worth quoting:

Since my student years Minkowski was my best, most dependable friend who supported me with all the depth and loyalty that was so characteristic of him. Our science, which we loved above all else, brought us together; it seemed to us a garden full of flowers. In it, we enjoyed looking for hidden pathways and discovered many a new perspective that appealed to our sense of beauty, and when one of us showed it to the other and we marveled over it together, our joy was complete. He was for me a rare gift from heaven and I must be grateful to have possessed that gift for so long. Now death has suddenly torn him from our midst. However, what death cannot take away is his noble image in our hearts and the knowledge that his spirit continues to be active in us.

# Nazi Mayhem

Finally, I turn to the depressing view of what Hitler and Stalin and their systems did to various mathematicians. Those who fell victim to the Nazis are numerous—I'll focus on two: Hausdorff and Schur.

# Hausdorff

Felix Hausdorff (1868–1942), an urbane Jew, had more of a name in literature and philosophy (where he used the pseudonym Paul Mongré) than in mathematics, especially prior to his moving from Leipzig to Bonn in 1910. There Study got him interested in geometry, and within a few years, he axiomitized topology, found the precursor to the Banach–Tarski paradox, and invented Hausdorff metric, measure, and dimension.

As the 1930s progressed, things got progressively worse: he was dismissed from his position in 1935. On January 25, 1942, expecting to be picked up for deportation to camps in the East, Hausdorff, his wife, and her sister took overdoses of barbiturates and died.

# Schur



Issai Schur (1875–1941) was a Jewish mathematician, for many years a professor in Berlin. His contributions to mathematics are well known. In the early 1930s, he turned down several invitations to leave Germany, thinking that this was Germany after all and they would come to their senses. In 1933, he was dismissed from his position, and in 1938, he was pressured to resign from the Prussian Academy. Bieberbach (he, Teichmüller, and Blaschke show you can be thoroughly evil and still do good mathematics) remarked: "I find it surprising that Jews are still members of academic commissions."

# Schur

The following story told by Shiffer illustrates the isolation and humiliation suffered by someone like Schur:

*"Schur told me that the only person at the Mathematical Institute in Berlin who was kind to him was Grunsky, then a young lecturer. Long after the war, I talked to Grunsky about that remark and he literally started to cry: 'You know what I did? I sent him a postcard to congratulate him on his sixtieth birthday. I admired him so much and was very respectful in that card. How lonely he must have been to remember such a small thing.'"*

# Schur

In 1939, he was allowed to leave Germany for Palestine, but the exit tax took his savings. He was unable to find a position in a country teeming with refugees and a single academic institution. Destitute, poor, and spiritually broken, he died in 1941.

# Fritz Noether

Fritz Noether (1884–1941) was the brother of Emmy Noether (1882–1935). Emmy is justly more celebrated than Fritz, but Fritz made one great discovery. In 1921, he realized certain singular integral equations were noncompact but had an integer invariant—their index. Thirty years later, in others' hands, this blossomed to the theory of Fredholm operators, custom-made for Gel'fand, Atiyah, and Singer ten years after that.

# Fritz Noether

Both Emmy and Fritz had German positions from which, as Jews, they were dismissed in 1934. Emmy went to the U.S. and died of cancer a year later.

Fritz went to Tomsk. In 1937, he was accused of being a German spy and imprisoned. In 1941, he was shot for anti-Soviet propaganda. In 1988, the Supreme Court of the Soviet Union officially exonerated him.

# Final Thoughts



I hope you've learned that our forefathers are fascinating as people and that you'll consider using Mr. Google and Ms. Wikipedia to look up the names you find on theorems.

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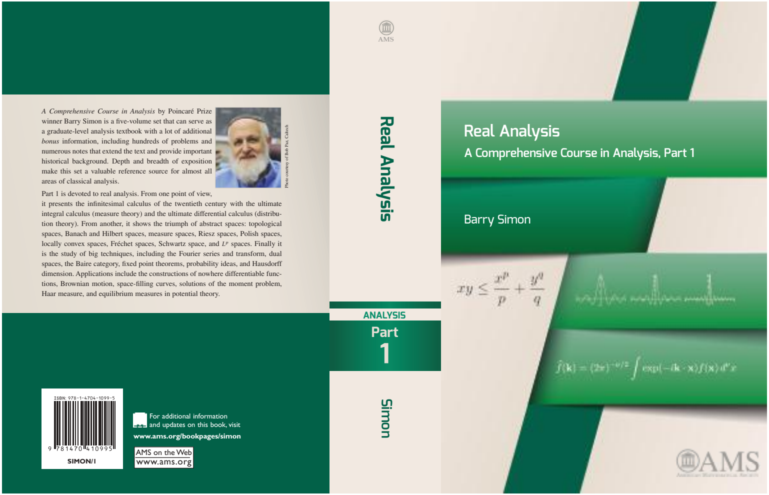


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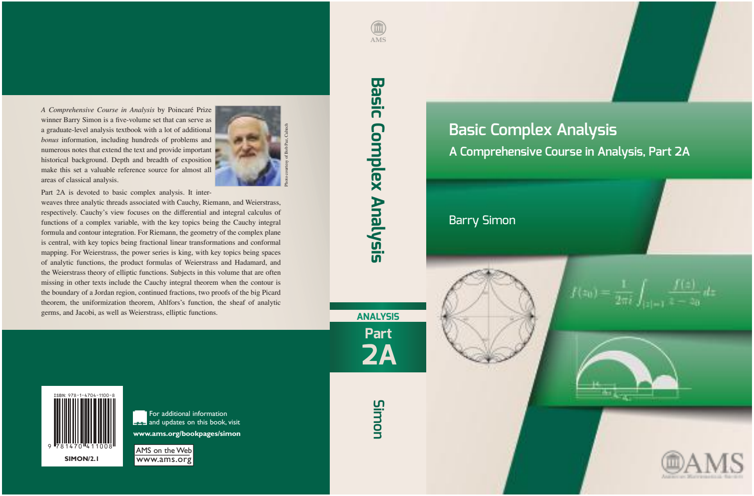


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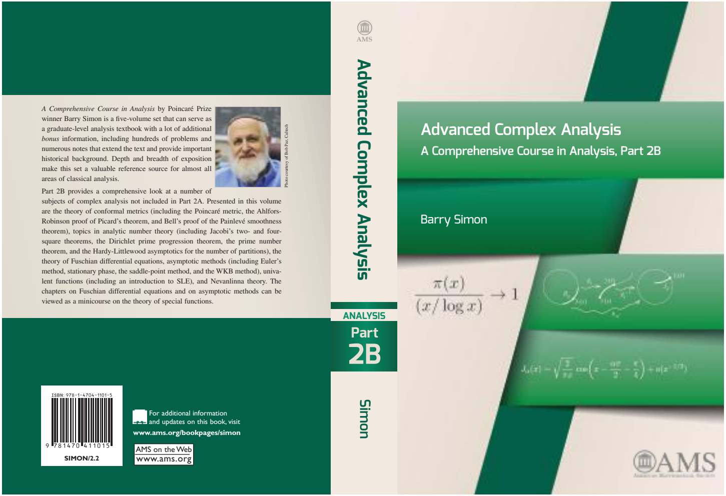


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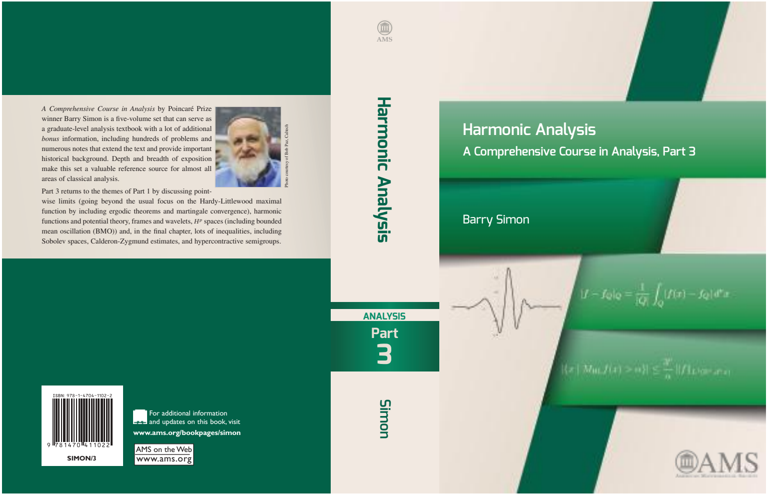


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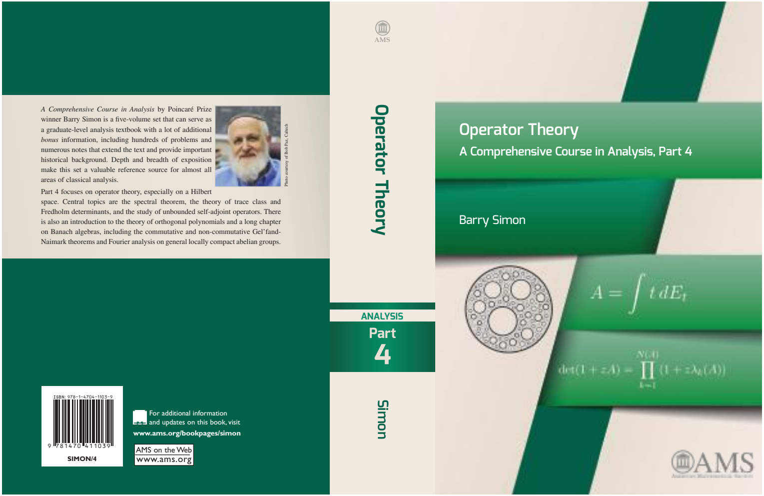


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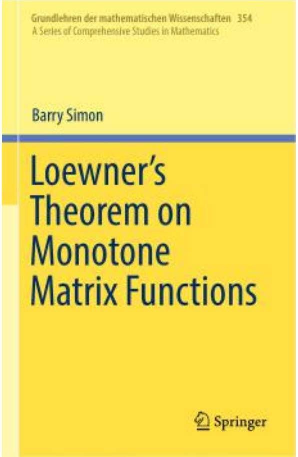


And tada, the latest book