Mini-Symposium on Analytic Number Theory and Applications

Titles and Abstracts

Linz, Monday April 1st, 2019

Normal Numbers with Digital Restrictions

Christoph Aistleitner (TU Graz, Austria)

Abstract: The concept or normality of numbers was introduced by Borel in 1909. Several constructions of normal numbers are known, while it is notoriously difficult to check whether a certain number is normal or not. In this talk we are concerned with the question how digital manipulations affect the normality of a number, and if there are normal numbers which satisfy certain prescribed digital restrictions. Based on joint work with Veronica Becher and Olivier Carton.

Prime and Square-free Values of Polynomials via the Circle Method and Rational Points

Kevin Destagnol (IST Vienna, Austria)

Abstract: We will present a generalisation of Schinzel's hypothesis and of the Bateman-Horn conjecture concerning prime values of systems of polynomials in one variable to the case of a integer form in many variables. In particular, we will establish in this talk that a polynomial in moderately many variables takes infinitely many prime (but also squarefree) values under some necessary assumptions. The proof will rely on Birch's circle method and will be achieved in 50% fewer variables than in the classical Birch setting. Moreover our result can be applied to study the Hasse principle and weak approximation for some normic equations. This is joint work with Efthymios Sofos.

Multiple Dirichlet Series with Arithmetical Coefficients on the Numerators

Kohji Matsumoto (Nagoya University, Japan)

Abstract: We discuss various types of multiple Dirichlet series with arithmetical coefficients on the numerators. The properties of those Dirichlet series surely depend on what kind of coefficients is on the numerators. Some of them can be continued meromorphically in the whole space, while some others have natural boundaries. One of the most interesting cases is that the coefficients are von Mangold functions. In this case there is a connection between the relevant Dirichlet series and Goldbach's problem. I will report the contents of my several papers, written jointly with various people, some are rather old, while some are quite new.

Geometric Properties of Arithmetic Dynamical Systems

László Mérai (RICAM, Austria)

Abstract: Consider the orbits of a transformation $x \mapsto \psi(x)$ associated with a rational function $\psi(X) \in \mathbb{F}_p(X)$ over a prime field $\mathbb{F}_p$. More precisely, for $u \in \mathbb{F}_p$, we consider the sequence

$$u_0 = u, \quad u_{n+1} = \psi(u_n),\ n = 0, 1, \ldots$$

as a dynamical system on $\mathbb{F}_p$ and study how far it propagates in N steps or how large is the group the first N elements generate. In fact, we study the quantities

$$L_{\Psi,u}(N) = \min_{v\in\mathbb{F}_p} \max_{0\le n\le N} |u_n - v|$$

and

$$G_{\Psi,u}(N) = \min_{v\in\mathbb{F}_p^*} \#\langle vu_n \ : \ n = 0, \ldots, N\rangle.$$

We are mostly interested in the case when N is small compare to the field size.

Minimizing GCD sums, Multiplicative Energy and Applications

Marc Munsch (TU Graz, Austria)

Abstract: In recent years, the question of maximizing GCD sums regained interest due to its firm link with large values of L-functions, leading for instance to the breakthrough improvement of Bondarenko and Seip concerning the maximum of $|\zeta(1/2+it)|$. In this talk, we address the counterpart problem of minimizing weighted GCD sums and show that it appears naturally in some applications. We consider as well a related optimization question regarding the usual multiplicative energy of a subset of the first N integers. We derive from our results some consequences for short character sums and non-vanishing of theta functions.

Sequences of Imaginary Parts of Riemann Zeta-Zeros and Their Distribution

Athanasios Sourmelidis (Würzburg University, Germany)

Abstract: It is well-known that if $\{\gamma_n\}_{n\in\mathbb{N}}$ is the sequence of the imaginary parts of the Riemann zeta-zeros, then the sequence $\{\alpha\gamma_n\}_{n\in\mathbb{N}}$ is uniformly distributed modulo 1 for every $\alpha\in\mathbb{R}$. Applying a theorem of Littlewood, we study the distribution of subsequences of $\{\gamma_n\}_{n\in\mathbb{N}}$ whose terms are well-spaced. This work is motivated by a problem in the universality of the Riemann zeta-function ζ:

If K is a compact subset of the right-half of the critical strip with connected complement, f a continuous non-vanishing function on K which is analytic in its interior and $\varepsilon>0$, can we find γ from $\{\gamma_n\}_{n\in\mathbb{N}}$ such that

$$\max_{s\in K}|\zeta(s+i\gamma)-f(s)|<\varepsilon?$$

We give some applications of our results regarding this problem. This is joint work with Jörn Steuding.

Recent results on the Value-Distribution of Hurwitz Zeta-Functions

Jörn Steuding (Würzburg University, Germany)

Abstract: In the 1970s Voronin discovered a remarkable universality property of the Riemann zeta-function, namely that certain shifts of the zeta-function inside the critical strip can approximate any admissible target function as good as we please. Since the class of admissible target functions is rather large (non-vanishing analytic functions defined on a compact subset of the open right half of the critical strip with connected complement), this property is called 'universality'. Around 1980 Gonek and (independently) Bagchi showed that the Hurwitz zeta-function with a rational or a transcendental parameter is universal too. In the talk we shall discuss recent research on the open case of an algebraic irrational parameter. This is joint work with Thanasis Sourmelidis.

Equidistribution, Van der Corput Sets and Exponential Sums

Robert Tichy (TU Graz, Austria)

Abstract: A set $H \subseteq \mathbb{Z}$ is called a van der Corput set if any sequence $(x_n)_{n\in\mathbb{N}}$ is equidistributed provided that the difference sequences $(x_{n+h} - x_n)_{n\in\mathbb{N}}$ are equidistributed for all $h \in H$. By van der Corput's difference theorem, $H = \mathbb{N}$ is a van der Corput set. This concept is related to sets of recurrence and difference sets and other concepts from additive combinatorics. We establish new results on sets of recurrence and van der Corput sets in

$\mathbb{Z}^k$ (i.e. for $\mathbb{Z}^k$-actions) which refine and unify some of the previous results obtained by Sarkőzy, Furstenberg, Kamae and Mèndes France, and Bergelson and Lesigne. Furthermore, we construct some new examples of such sets involving prime numbers. This involves new bounds for exponential sums containing generalized polynomials of the form

$$f(x) = \sum_{j=1}^{m} \alpha_j x^{\theta_j}$$

where $0 < \theta_1 < \theta_2 < \cdots < \theta_m$, α_j are non-zero reals and at least one α_j is irrational if all $\theta_j \in \mathbb{N}$. Furthermore, we apply this method to diophantine inequalities involving prime numbers p. As a special result we obtain

$$\min_{1 \le p \le N} || \, \xi \lfloor f(p) \rfloor \, || \ll N^{-\eta}$$

where ξ is a real number, N a sufficiently large positive integer and $||.||$ denotes the distance to the nearest integer, $\lfloor . \rfloor$ the floor function and $\eta > 0$ a suitable exponent. This is recent joint work with Manfred Madritsch.

Some Applications of Character Sums

Arne Winterhof (RICAM, Austria)

Abstract: Character sums are important tools in the theory of finite fields and have many application areas including coding theory, wireless communication, Monte Carlo methods, pseudorandom number generation, analysis of algorithms, and quantum physics. The talk contains a collection of applications including some classical and well-known ones such as the construction of Hadamard matrices as well as more recent and maybe less known ones as the analysis of nonlinear pseudorandom numbers and mutually unbiased bases for measuring quantum states.

On the Existence of S-Diophantine Quadruples

Volker Ziegler (Salzburg University, Austria)

Abstract: Let S be a set of primes. We call an m-tuple $(a_1, \dots, a_m)$ of distinct, positive integers S-Diophantine, if for all $i \neq j$ the integers $s_{i,j} := a_i a_j + 1$ have only prime divisors coming from the set S, i.e. if all $s_{i,j}$ are S-units. In this talk, we show how to prove that in the case that $S = \{3, q\}$ no S-Diophantine quadruples (i.e. $m = 4$) exists. Furthermore we show that for all pairs of primes (p, q) with $p < q$ and $p \equiv 3 \mod 4$ no $\{p, q\}$-Diophantine quadruples exist, provided that (p, q) is not a Wieferich prime pair. This will lead us to questions on the existence of infinitely many Wieferich prime pairs and related questions.

Enjoy your time in Linz!

We wish you will have an interesting and fruitful meeting!