Faster sampling of random variables

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Inversion Method

Fast algorithms invert the cdf (approximately) and sample $U \sim \mathcal{U}(0,1)$ to get

$$X = F^{-1}(U)$$

Also works easily for QMC methods.

- What about sampling random processes?
- Random variables without a cdf in a simple form?
- Is there a universal way to find a good approximation $\widehat{F^{-1}}$ for F^{-1}?

Training by simulation

Cut expensive simulation costs

- (Pre-)Train a sampling method (neural network approx. $\widehat{F^{-1}}$) with expensive simulations.
- Use the method to generate samples quickly in applications.

How to train?

- Solve $F(\widehat{F^{-1}(U)}) = U$
- Compare empirical cdf, icdf, ...
- Compare moments or characteristic function
- Statistical tests

Lévy area

For higher order SDE approximation (Milstein order 1 compared order 1/2 of Euler scheme) we need to sample Brownian increments dW and Lévy areas $A(h)$, where

$$A_{ij}(h) = \frac{1}{2} \int_t^{t+h} \int_t^s dW^i dW^j - dW^j dW^i$$

conditional char. func $\varphi_{A_{ij}(h)}$, conditional on dW^i, dW^j known, CDF not in closed form

- Train by sampling Brownian bridge, Fourier methods...?
- Infer tail-behaviour from $\varphi_{A_{ij}(h)}$? (tail is hard to train)
- How to sample from higher order iterated integrals?

Convergence

After M simulations/training iterations:
For an approximation $\widehat{F_M^{-1}}$ of the RV X

- Which errors should we analyze?
- Which loss function to choose for training?
- How good is the sampling method after M training iterations?
- For example, what does the emp. char. function tell us

$$loss = \sup_t \|\mathrm{E}[e^{itX}] - \frac{1}{N}\sum_{n=1}^{N} e^{it\widehat{F_M^{-1}}(U_n)}\|$$

for $U_n \sim \mathcal{U}(0,1)$

Thank you for your attention.