Aicke Hinrichs

Integration of trig. polynomials

$F_1 = \mathrm{span}\{1, \cos 2\pi x, \sin 2\pi x\}$ $[0,1]$

form an orthonormal basis

$F_d = F_1 \otimes \cdots \otimes F_1$ Hilbert space product

dim 3^d

$$K_d(x,y) = \prod_{i=1}^{d} [1 + \cos 2\pi(x_i - y_i)]$$

$$I(f) = \int_{[0,1]^d} f(x)\,dx \qquad \|I\| = 1$$

$$Q_n(f) = \sum_{i=1}^{n} c_i f(x_i)$$

$$e^{wor}(Q_n)^2 = \sup_{\|f\| \le 1} |I(f) - Q_n(f)|^2$$

$$= 1 - 2\sum_{j=1}^{n} c_j + \sum_{j,k=1}^{n} c_j c_k K_d(x_j, x_k)$$

$$e^{wor}(Q_n)^2 \geq \max(1 - n 2^{-d}, 0) \quad \| \text{ Novak}$$

exp. many pts needed - Curse

$$\boxed{c_i \geq 0}$$

PROBLEM Prove this also for

$C_i \in \mathbb{R}$.

CONJECTURE 1 $x_1, \dots, x_n \in \mathbb{R}^d$

$$\left(\prod_{i=1}^{d} \frac{1+\cos(x_{j,i} - x_{k,i})}{2} - \frac{1}{n} \right)_{j,k=1}^{n}$$

is positive definite. C_i

CONJECTURE 2 $f : \mathbb{R}^d \to \mathbb{R}$ bounded, positive, continuous, positive definite

$\boxed{f(0) = 1}$ $\qquad \hat{f} \geq 0$

$$\left(f(x_j - x_k) - \frac{1}{n} \right)_{j,k=1}^{n}$$

is positive semi-definite.