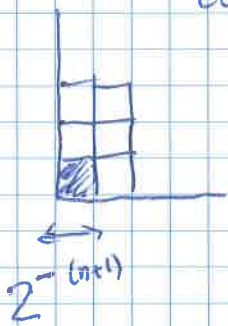


Sharpness of signed SBC

$$\exists \varepsilon_R = \pm 1 : \left\| \sum_{|R|=2^{-n}} \varepsilon_R h_R \right\|_\infty \approx n^{d/2}$$

$$\varepsilon_R = \text{iid } \pm 1$$

cubes Q_1, Q_2, \dots, Q_M $M = 2^{(n+1)d}$



$$X_k = \sum \varepsilon_R h_R |_{Q_k}$$

X_k - a sum of $\approx n^{d-1}$ iid ± 1

$$\mathbb{E} |X_k| \approx n^{d/2}$$

$$\mathbb{E} \max_{1 \leq k \leq M} |X_k| \approx \frac{\mathbb{E} |X_k|}{\sqrt{\log M}}$$

HOEFFDING INEQUALITY

$$\mathbb{P}(|X_k| > t) \leq 2 \exp\left(-\frac{t^2}{cn^{d-1}}\right)$$

$$\mathbb{E}(\exp(tX)) \leq \exp(cn^{d-1} t^2)$$

Jensen

$$\exp(t \mathbb{E} \max_{1 \leq k \leq M} |X_k|) \leq \mathbb{E} \max_{1 \leq k \leq M} \exp(t|X_k|)$$

$$\leq \mathbb{E} \sum_{k=1}^M \exp(t|X_k|)$$

$$\leq 2M \exp(c n^{d-1} t^2)$$

$$\mathbb{E} \max_{1 \leq k \leq M} |X_k| \leq \frac{1}{t} \cdot \log 2M + c \cdot n^{d-1} \cdot t$$

$$t^2 = \frac{\log 2M}{c n^{d-1}} \quad t = \frac{\sqrt{\log 2M}}{c' n^{\frac{d-1}{2}}}$$

$$\mathbb{E} \max_{1 \leq k \leq M} |X_k| \lesssim \sqrt{\log(2M)} \cdot n^{\frac{d-1}{2}}$$

$M = 2^{(n+1)d}$ $\log M \approx n$	}	<p>in our case</p> $\mathbb{E} \ \sum \varepsilon_k h_k\ _\infty \lesssim$ $\lesssim \sqrt{n} \cdot n^{\frac{d-1}{2}} = n^{d/2}$
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(Exponential) Orlicz spaces.

$$\psi: \mathbb{R}_+ \rightarrow \mathbb{R}_+ \quad \psi \geq 0 \quad \psi(0) = 0 \quad \text{convex}$$

$$\|f\|_{L^\psi} = \inf \left\{ K > 0 : \int_{\Omega} \psi\left(\frac{|f(x)|}{K}\right) dx \leq 1 \right\}$$

$$\psi(x) = x^p \quad p \geq 1 \quad L^\psi = L^p$$

$$\psi(x) = e^{x^\alpha} - 1 : \exp(L^\alpha) \text{ - exponential Orlicz space}$$

$\alpha = 2$ subgaussian functions

$$\|f\|_{\exp(L^\alpha)} \stackrel{\int f = 0}{\approx} \sup_{t > 0} \frac{\log P(|f| > t)}{-t^{1+\alpha}}$$

$$\approx \sup_{p > 1} p^{-\frac{1}{\alpha}} \cdot \|f\|_p$$

$$L^\infty \subset \exp(L^\alpha) \subset L^p$$

$$L^1 \supset L(\log L)^p \supset L^p \quad p > 1$$

$$F = \sum_{|R|=2^{-n}} \varepsilon_R h_R$$

$$SF = \text{const} \approx N^{\frac{d-1}{2}}$$

Littlewood - Paley (hyperbolic)

$$\|F\|_p \lesssim p^{\frac{d-1}{2}} \|SF\|_p$$

$$\leq p^{\frac{d-1}{2}} N^{\frac{d-1}{2}}$$

$$\|F\|_{\exp(L^{\frac{2}{d-1}})} \lesssim N^{\frac{d-1}{2}}$$



$$\|D_N\|_{L \cdot (\log L)^{\frac{d-1}{2}}} \gtrsim (\log N)^{\frac{d-1}{2}}$$

$$\psi(x) = x \cdot \underbrace{(1 + \log x)}_{|x+1|}^{\frac{d-1}{2}}$$

$\begin{matrix} 1 \text{ dim} & & d-1 \text{ dim} \\ \downarrow & & \downarrow \\ X = (x', \bar{x}) \end{matrix}$

Theorem (Karstlidis, 15)

$R = R' \times \tilde{R}$

$\left\| \sum_{|R|=2^n} \epsilon_R h_R \right\|_{\infty} \approx N^{d/2}$

$\epsilon_R = \epsilon_{R'} \cdot \epsilon_{\tilde{R}}$

Fix \bar{x}

1-dim

$\epsilon_{R'}$

$\left\| \sum \epsilon_R h_R \right\|_{L^\infty(x')} = \left\| \sum_{|R'| \geq 2^{-n}} \left[\sum_{|\tilde{R}|} \epsilon_{\tilde{R}} h_{\tilde{R}}(\bar{x}) \right] \epsilon_{R'} h_{R'}(x') \right\|_{\infty}$

$|\tilde{R}| = \frac{2^{-n}}{|R'|}$

$= \left\| \sum_{k=0}^n \sum_{|R'|=2^{-k}} \left[\sum_{|\tilde{R}|=2^{-(n-k)}} \epsilon_{\tilde{R}} h_{\tilde{R}}(\bar{x}) \right] \epsilon_{R'} h_{R'}(x') \right\|_{\infty}$

iid $\neq 1$
for $k=0, 1, \dots, n$

$= \sum_{k=0}^n \left| \sum_{|\tilde{R}|=2^{-k}} \epsilon_{\tilde{R}} h_{\tilde{R}}(\bar{x}) \right|$

$$\| \sum \alpha_k \overset{\text{iid } \pm 1}{r_k} \|_{\infty} = \sum |\alpha_k|$$

$$\| \sum \varepsilon_R h_R \|_{\infty} = \left\| \sum_{k=0}^n \left| \sum_{|\tilde{R}|=2^{-k}} \varepsilon_{\tilde{R}} h_{\tilde{R}}(\tilde{x}) \right| \right\|_{L^{\infty}(\tilde{x})}$$

$$\geq \left\| \sum | \dots | \right\|_{L^1(\tilde{x})}$$

$$= \sum_{k=0}^n \left\| \sum_{|\tilde{R}|=2^{-k}} \varepsilon_{\tilde{R}} h_{\tilde{R}}(\tilde{x}) \right\|_{L^1}$$

$$\approx \sum_{k=0}^n \left\| \dots \right\|_{L^2}$$

$$\approx \sum_{k=0}^n k \frac{d-2}{2} \approx n^{\frac{d-2}{2}} \cdot n$$

$$= n^{d/2}$$



Sidon

$$1) \left\| \sum a_k \sin(2\pi 2^k x) \right\|_{\infty} \approx \sum |a_k|$$

$$2) \|f\|_{\infty} \approx \|f\|_2$$



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Skriganov 2014

For any $P_N \in [0,1]^d$

there exist a digit shift T

$P_N \oplus T$ satisfies

$$\|E_T\| \|\mathcal{D}_{P_N \oplus T}\|_\infty \approx (\log N)^{d/2}$$

Gilbert-Varshamov bound

C - binary code of length m : $C \subset \{0,1\}^m$

Hamming distance:

$$d(x, y) = \#\{1 \leq i \leq m : x_i \neq y_i\}$$

Lemma Let C be the maximal binary code of length m with min. Hamming distance d .

Then

$$\#C \geq \frac{2^m}{\sum_{k=0}^d \binom{m}{k}}$$

$$B_H(x, d) = \{y \in \{0,1\}^m : d_H(x, y) \leq d\}$$

$$\#B_H = \sum_{k=0}^d \binom{m}{k}$$

$$\#C \cdot \#B_H(x, d) \geq 2^m$$