$X \subseteq S^d$ $\quad$ ($d=2$)

spherical t-designs

normalized

$\deg(p) \le t$ $\quad \frac{1}{\#X} \sum_{x \in X} p(x) = \int_{S^d} p(y)\, d\sigma_d(y)$

D-G-S: $\underline{d=2}$ $\quad \approx \left(\frac{t}{4}\right)^2 1.07 \quad \approx \frac{t^2}{2}$

$F(x) = \sum_{n=0}^{\infty} \underset{\geq}{a_n} P_n(x)$ ← Legendre-Polyn.

$\underline{a_n < 0} \quad n > t$

$F(x) \geq 0 \quad -1 \leq x \leq 1$

$$\sum_{n=0}^{\infty} a_n \underbrace{\sum_{x,y \in X} P_n(\langle x,y \rangle)}_{=0,\ 1 \leq n \leq t} \geq \sum_{x,y \in X} F(\langle x,y \rangle) \geq \#X \cdot F(1)$$

$$(\#X)^2 \cdot a_0 = (\#X)^2 \cdot \frac{1}{2} \int_{-1}^{1} F(x)\, dx$$

$$\#X \geq \frac{F(1)}{\frac{1}{2}\int_{-1}^{1} F(x)\, dx}$$

2/2

II f, g

$(\ell-n)(\ell+n+1)$

$$f * g = \sum_{n=0}^{\infty} \hat{f}_n \cdot \hat{g}_n \, P_n(x)$$

$$g = Lf + \ell(\ell+1) f$$

$$L = ((1-x^2) f')'$$

$$\int (f * g)\, dx = c \cdot \left(\int f \, dx \right)^2 \overset{!}{=} 1$$

$$f * \quad f * g(1) = \int_{-1}^{1} f(x) \cdot g(x)\, dx$$