

Sum-product problem

$$A \subseteq \mathbb{R}, \mathbb{Z}, \mathbb{F}_q$$

$$A + A = \{a + b : a, b \in A\}$$

$$A \cdot A = \{ab : a, b \in A\}$$

Consider $A \cdot (A + A) = \{abc + c : a, b, c \in A\}$

Conjecture 1: For any $A \subseteq \mathbb{R}$

$$|A \cdot (A + A)| \geq |A|^2$$

We know $|A \cdot (A + A)| \geq |A|^{\frac{3}{2}}$

Fact: $|A \cdot (A + A)| \geq |A \cdot A|$

What about $|A \cdot (A + A)| \geq |A + A|$?

Conjecture 2: $A \subseteq \mathbb{R}$

$$|A \cdot (A + A)| \geq |A + A|$$

There exists $A \subseteq \mathbb{F}_p$ s.t.

$$|A \cdot (A + A)| = p - 1, \quad |A + A| = p$$

Conjecture 3:

$$|AA+A| \geq |A/A|$$



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